Workshop

Stochastic modelling: Foundation and Statistics

August 16, 2021

Organisers: Zhigang Bao (HKUST), Jianfneg Yao (HKU), Ke Zhu (HKU)

Abstracts

9:30–10:15 *Supercritical Spatial SIR Epidemics: Spreading Speed and Herd Immunity*

Xinghua Zheng
Department of Information Systems, Business Statistics and Operations Management, Hong Kong University of Science and Technology

We study supercritical spatial SIR epidemics on $\{1, 2, \ldots, N\}^{\mathbb{Z}^2}$, where each site in $\mathbb{Z}^2$ represents a village and N stands for the village size. We establish several key asymptotic results as $N \to \infty$. In particular, we show that the epidemic spread out linearly in all directions, and derive an explicit formula for the spreading speed. Moreover, we prove that the ultimate proportion of infection converges to a number that is constant over space and find its explicit value. An important message is that if there is no vaccination, then the ultimate proportion of population who will be infected can be *much higher* than the vaccination proportion needed in order to prevent sustained spread of the infection.

The talk is based on joint work with Qingsan Zhu.

10:15–11:00 *No exceptional words for site percolation on $\mathbb{Z}^3$*

Pierre Nolin
Department of Mathematics, City University of Hong Kong

Bernoulli percolation is a model for random media introduced by Broadbent and Hammersley in 1957. In this process, each vertex of a given graph is occupied or vacant, with respective probabilities p and $1-p$, independently of the other vertices (for some parameter p). It is arguably one of the simplest models from statistical mechanics displaying a phase transition as the parameter p varies, i.e. a drastic change of behaviour at some critical value p_c, and it has been widely studied.

Benjamini and Kesten introduced in a 1995 paper the problem of embedding infinite binary sequences into a Bernoulli percolation configuration, known as percolation of words. We give a positive answer to their Open Problem 2, which had stayed widely open since then: for percolation on $\mathbb{Z}^3$ with parameter $p = 1/2$, we prove that almost surely, all words can be embedded. We also discuss various extensions of this result. This talk is based on a joint work with Augusto Teixeira (IMPA) and Vincent Tassion (ETH Zürich).

11:00–11:30 Coffee break

11:30–12:00 *Critical branching random walks, branching capacity and branching interlacements*
Qingsan Zhu
Institute for advanced study, Hong Kong University of Science and Technology

We discuss critical branching random walks and introduce new concepts for critical branching random walks: branching capacity and branching interlacements. By introducing these concepts, we can obtain analogues of various classical results for random walks, in the setting of critical branching random walks: the exact asymptotics for the visiting probability, the local limit of branching random walks in tori, etc.

12:00–12:30 *Eigenvalue distributions of large auto-covariance matrices*
Wangjun Yuan
Department of Mathematics, The University of Hong Kong

In this work, we establish a limiting distribution for eigenvalues of a class of auto-covariance matrices. The same distribution has been found in the literature for a regularized version of these auto-covariance matrices. The original non-regularized auto-covariance matrices are non invertible which introduces supplementary difficulties for the study of their eigenvalues through Girko's Hermitization scheme. The key result in this paper is a new polynomial lower bound for a specific family of least singular values associated to a rank-defective quadratic function of a random matrix with independent and identically distributed entries. Another improvement in the paper is that the lag of the auto-covariance matrices can grow to infinity with the matrix dimension.

This is a joint work with Jianfeng Yao.

12:30–14:00 Lunch break

14:00–14:45 *High-Dimensional Low-Rank Tensor Autoregressive Time Series Modelling*
Guodong Li
Department of Statistics and Actuarial Science, The University of Hong Kong

Modern technological advances have enabled an unprecedented amount of structured data with complex temporal dependence, urging the need for new methods to efficiently model and forecast high-dimensional tensor-valued time series. This paper serves as the first thorough attempt in this direction via autoregression. By considering a low-rank Tucker decomposition for the transition tensor, the proposed tensor autoregression can flexibly capture the underlying low-dimensional tensor dynamics, providing both substantial dimension reduction and meaningful dynamic factor interpretation. For this model, we introduce both low-dimensional rank-constrained estimator and high-dimensional regularized estimators, and derive their asymptotic and non-asymptotic properties. In particular, a novel convex regularization approach, based on the sum of nuclear norms of square matricizations, is proposed to efficiently encourage low-rankness of the coefficient tensor. A truncation method is further introduced to consistently select the Tucker ranks. Simulation experiments and real data analysis demonstrate the advantages of the proposed approach over various competing ones.

14:45–15:30 *Nonparametric Inference for Network Moments*
Dong Xia
Department of Mathematics, Hong Kong University of Science and Technology

Network method of moments is an important tool for nonparametric network inference. However, there has been little investigation on accurate descriptions of the sampling distributions of network moment statistics. We present the first higher-order accurate approximation to the sampling CDF of a studentized network moment by Edgeworth expansion. Our assumptions match the minimum requirements in related

literature. For sparse networks, our theory shows that our empirical Edgeworth expansion and a simple normal approximation both achieve the same gradually depreciating Berry-Esseen type bound as the network becomes sparser. This result also significantly refines the best previous theoretical result. We showcase three applications of our results in network inference. We prove, to our knowledge, the first theoretical guarantee of higher-order accuracy for some network bootstrap schemes, and moreover, the first theoretical guidance for selecting the sub-sample size for network sub-sampling. We also derive a one-sample test and the Cornish-Fisher confidence interval for a given moment with higher-order accurate controls of confidence level and type I error, respectively.

15:30–16:00 Coffee break

16:00-16:45 *Autoregressive networks*
Bingyan Jiang
Department of Applied Mathematics, The Hong Kong Polytechnic University

We propose a first-order autoregressive model for dynamic network processes in which edges change over time while nodes remain unchanged. The model depicts the dynamic changes explicitly. It also facilitates simple and efficient statistical inference such as the maximum likelihood estimators which are proved to be (uniformly) consistent and asymptotically normal. The model diagnostic checking can be carried out easily using a permutation test. The proposed model can apply to any network processes with various underlying structures but with independent edges. As an illustration, an autoregressive stochastic block model has been investigated in depth, which characterizes the latent communities by the transition probabilities over time. This leads to a more effective spectral clustering algorithm for identifying the latent communities. Inference for a change point is incorporated into the autoregressive stochastic block model to cater for possible structure changes. The developed asymptotic theory as well as the simulation study affirms the performance of the proposed methods. Application with three real data sets illustrates both relevance and usefulness of the proposed models.

This is a joint work with Jialiang Li and Qiwei Yao.

16:45–17:15 *Implied conditional moments by Cornish-Fisher expansion and their applications*
Ningning Zhang
Department of Statistics and Actuarial Science, The University of Hong Kong

This paper proposes a novel simple method to learn the conditional mean, variance, skewness, and kurtosis by using the classical Cornish-Fisher expansion. The existing methods usually specify certain parametric models to estimate the conditional moments, whereas our method provides an easy-to-implement non-parametric way to estimate the so-called implied conditional moments, based on a sequence of estimated conditional quantiles. Some regression tests are proposed to check the validity of our implied conditional moments. Simulations show that the implied conditional moments could be good proxies for their unobserved counterparts, and they exhibit robust performances across the choices of quantile estimation method and quantile level. As three important applications, the implied conditional moments unveil some new volatility-in-mean effects, news impact functions, and interactive effects among the conditional moments.