For favour of posting

DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE
THE UNIVERSITY OF HONG KONG

50th Anniversary Seminar Series

Professor Marc HALLIN

ECARES and Département de Mathématique
Université libre de Bruxelles,
Belgium

will give a talk
entitled

ON DISTRIBUTION AND QUANTILE FUNCTIONS IN $\mathbb{R}^d$
A MEASURE-TRANSPORTATION APPROACH

Abstract

Unlike the real line, the real space $\mathbb{R}^d$, $d \geq 2$ is not "naturally" ordered. As a consequence, such fundamentals univariate concepts as quantile and distribution functions, ranks, signs, all order-related, do not straightforwardly extend to the multivariate context. Since no universal pre-existing order exists, each distribution, each data set, has to generate its own: the rankings behind any sensible concept of distribution or quantile function, ranks, or signs, inherently will be distribution-specific and, in empirical situations, data-driven.

Many proposals have been made in the literature for such orderings—all extending some aspects of the univariate concepts, but failing to preserve the essential properties that make classical rank-based inference a major inferential tool in the analysis of semiparametric models where the density of some underlying noise remains unspecified: (i) exact distribution-freeness, and (ii) asymptotic semiparametric efficiency, see Hallin and Werker (Bernoulli 2003).

Distribution and quantile functions and their empirical counterparts (ranks, signs, and empirical quantiles) are well understood and well developed for univariate observations. We start by establishing the close connection between those classical concepts and measure transportation results, showing that they actually reduce to uniquely defined gradients of convex functions mapping a distribution to the uniform over the unit ball.

That fact, along with the powerful result by McCann (1995) on the existence and unicity of such gradients of convex functions in $\mathbb{R}^d$ is then exploited to define fully general concepts of distribution and quantile functions (along with their empirical counterparts—called the *Monge-Kantorovich ranks and signs*) coinciding, in the univariate setting, with the traditional concepts, and enjoying under completely unspecified (absolutely continuous) d-dimensional distributions, the essential properties ((i) and (ii)) that make traditional rank-based inference an essential tool in semiparametric inference.

Based on Chernozhukov, V., A. Galichon, M. Hallin, and M. Henry (2017) Monge-Kantorovich depth, quantiles, ranks, and signs, *Annals of Statistics* **45**, 223-256.

on

Wednesday, June 14, 2017

(Refreshments will be served from 2:15 p.m. outside Room 301 Run Run Shaw Building)

2:30 p.m. – 3:30 p.m.

at

Room 301, Run Run Shaw Building

Visitors Please Note that the University has limited parking space. If you are driving please call the Department at 3917 2466 for parking arrangement.

ALL INTERESTED ARE WELCOME